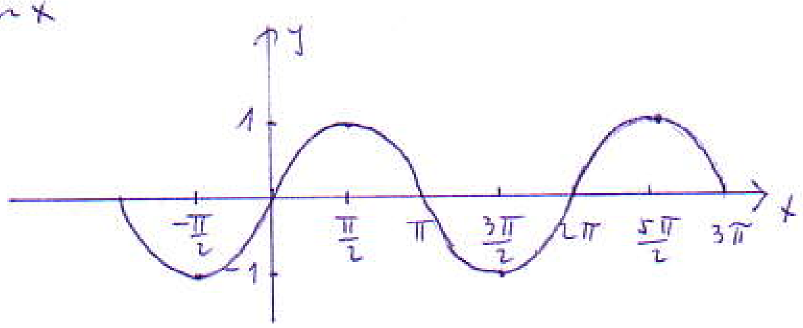
11. évfolyam

Trigonometrikus egyenletek

$f(x) = \sin x$



ÉT: $x \in \mathbb{R}$

ÉK: $f(x) \in [-1; 1]$

$\sin x = 1$

$x = \frac{\pi}{2} + 2k\pi \quad k \in \mathbb{Z}$

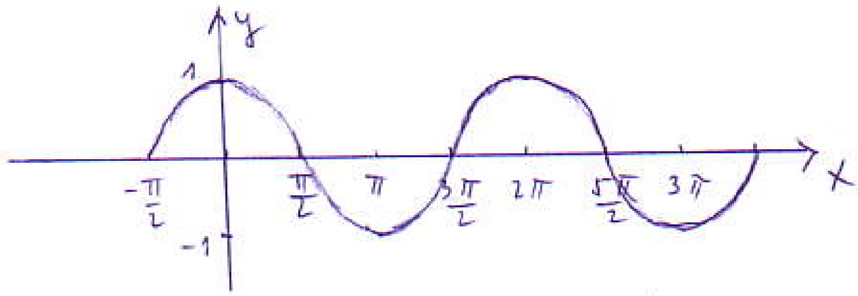
$\sin x = -1$

$x = \frac{3\pi}{2} + 2k\pi$

$\sin x = 0$

$x = k\pi$

$g(x) = \cos x$



ÉT: $x \in \mathbb{R}$

ÉK: $g(x) \in [-1; 1]$

$\cos x = 1$

$x = 2k\pi \quad k \in \mathbb{Z}$

$\cos x = -1$

$x = \pi + 2k\pi$

$\cos x = 0$

$x = \frac{\pi}{2} + k\pi$

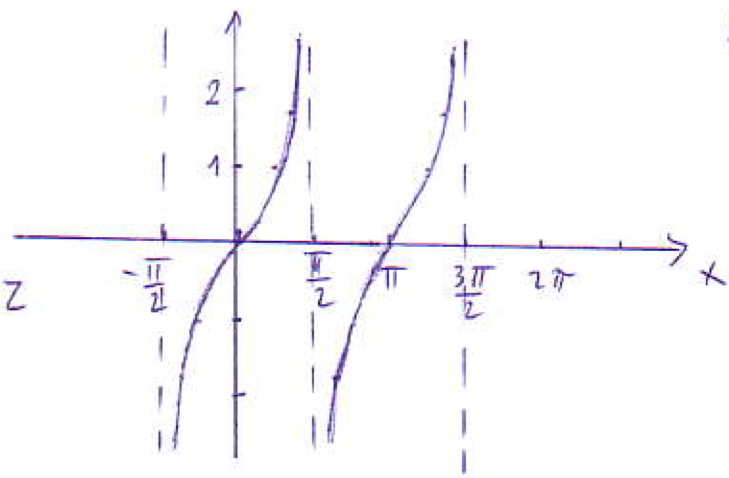
$h(x) = \operatorname{tg} x$

$\operatorname{tg} x = \frac{\sin x}{\cos x}$

$\cos x \neq 0$

$x \neq \frac{\pi}{2} + k\pi$

$k \in \mathbb{Z}$



ÉT: $x \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi\}$

ÉK: $h(x) \in \mathbb{R}$

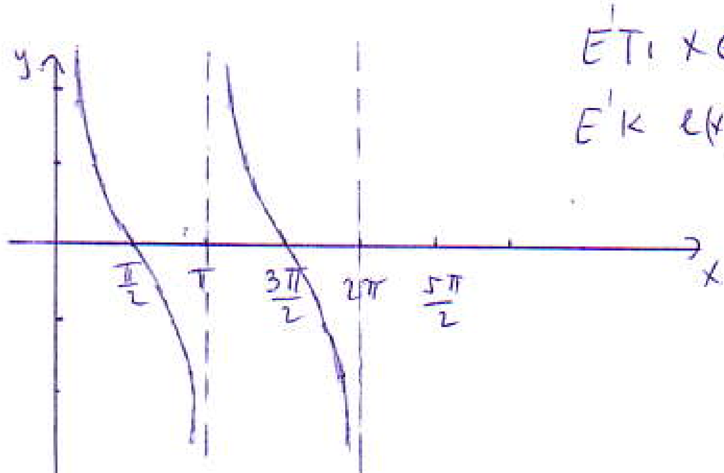
$l(x) = \operatorname{ctg} x$

$\operatorname{ctg} x = \frac{\cos x}{\sin x}$

$\sin x \neq 0$

$x \neq k\pi$

$k \in \mathbb{Z}$



ÉT: $x \in \mathbb{R} \setminus \{k\pi\}$

ÉK: $l(x) \in \mathbb{R}$

Típuspéldák

I. a) $\sin x = \frac{1}{2}$

$x_1 = 30°\cdot\frac{\pi}{180} + 2k\pi \quad k\in Z$

$x_2 = 150\cdot\frac{\pi}{180} + 2l\pi \quad l\in Z$

b) $\sin x = -\frac{1}{2}$

$x' = 30°$

$x_1 = (\underbrace{180°+30°}_{210°})\cdot\frac{\pi}{180} + 2k\pi \quad k\in Z$

$x_2 = (360°-30°)\cdot\frac{\pi}{180} + 2l\pi \quad l\in Z$

c) $\cos x = \frac{\sqrt{2}}{2}$

$x_1 = 45°\cdot\frac{\pi}{180°} + 2k\pi \quad k\in Z$

$x_2 = (360-45°)\frac{\pi}{180°} + 2l\pi \quad l\in Z$

d) $\operatorname{tg} x = 1$

$x = 45°\cdot\frac{\pi}{180°} + k\pi$

II. a) $\sin\left(2x+\frac{\pi}{3}\right) = \frac{1}{2}$

első síknegyed $2x + \frac{\pi}{3} = 30°\frac{\pi}{180} + 2k\pi \;/ -\frac{\pi}{3}$

$2x = \frac{\pi}{6} - \frac{\pi}{3} + 2k\pi$

$2x = \frac{\pi-2\pi}{6} + 2k\pi$

$2x = -\frac{\pi}{6} + 2k\pi \;/ :2$

$x = -\frac{\pi}{12} + k\pi$

második síknegyed

$2x + \frac{\pi}{3} = 150°\frac{\pi}{180} + 2k\pi \;/ -\frac{\pi}{3}$

$2x = \frac{5\pi}{6} - \frac{\pi}{3} + 2k\pi$

$2x = \frac{5\pi-2\pi}{6} + 2k\pi$

$2x = \frac{3\pi}{6} + 2k\pi$

$x = \frac{3}{12}\pi + k\pi$

b) $\cos 2x = \frac{1}{2}$

első síknegyed $2x = 60°\frac{\pi}{180} + 2k\pi \;/ :2$

$x_1 = \frac{\pi}{6} + k\pi$

negyedik síknegyed $2x = 300\cdot\frac{\pi}{180} + 2l\pi$

$x = \frac{5}{6}\pi + l\pi$

c) $2\sin x = -\sqrt{2}$

$\sin x = -\frac{\sqrt{2}}{2}$

$x' = 45°$

$x_1 = (180°+45°)\frac{\pi}{180°} + 2k\pi$

$x_2 = (360°-45°)\frac{\pi}{180} + 2l\pi$

III. a) $2\sin^2 x - 5\sin x + 2 = 0$

$a = 2$
$b = -5$
$c = 2$

$\sin x_{1,2} = \frac{5 \pm \sqrt{25-4\cdot 2\cdot 2}}{4} = \frac{5\pm 3}{4} \begin{cases} 2 \\ \frac{2}{4} = \frac{1}{2} \end{cases}$

$\sin x = 2$

Nincs megoldás

$\sin x = \frac{1}{2}$

$x_1 = 30°\cdot\frac{\pi}{180} + 2k\pi$

$x_2 = 150\cdot\frac{\pi}{180°} + 2l\pi$

b) $\operatorname{tg} x - \operatorname{ctg} x = 1$ $\quad x \neq \frac{\pi}{2} + k\pi \quad k \in Z$

$\operatorname{tg} x - \frac{1}{\operatorname{tg} x} = 1 \quad / \operatorname{tg} x$ $\quad x \neq k\pi$

$\operatorname{tg}^2 x - 1 = \operatorname{tg} x$

$\operatorname{tg}^2 x - \operatorname{tg} x - 1 = 0$

$\operatorname{tg} x_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$ $\quad \frac{1+\sqrt{5}}{2}$; $\frac{1-\sqrt{5}}{2}$

$\operatorname{tg} x = \frac{1+\sqrt{5}}{2}$ $\quad \operatorname{tg} x = \frac{1-\sqrt{5}}{2}$

$x = 58{,}3° + k\pi = 58{,}3 \cdot \frac{\pi}{180} + k\pi$ $\quad x = -31{,}7° + k\pi = -31{,}7° \cdot \frac{\pi}{180} + k\pi$

IV. $\cos^2 x - \cos x = \sin^2 x$

felhasználjuk a $\sin^2 x + \cos^2 x = 1$ azonosságot!

$\sin^2 x = 1 - \cos^2 x$

$\cos^2 x - \cos x = 1 - \cos^2 x$

$2\cos^2 x - \cos x - 1 = 0$

$a = 2$
$b = -1$
$c = -1$

$\cos x_{1,2} = \frac{1 \pm \sqrt{1 - 4 \cdot 2 \cdot (-1)}}{4} = \frac{1 \pm \sqrt{9}}{4} = \frac{1 \pm 3}{4}$ $\quad 1$; $-\frac{2}{4} = -\frac{1}{2}$

$\cos x = 1$

$x = 2k\pi \quad k \in Z$

$\cos x = -\frac{1}{2}$

$x' = 60°$

$x_1 = (180° - 60°)\frac{\pi}{180} + 2l\pi \quad l \in Z$

$x_2 = (180° + 60°)\frac{\pi}{180} + 2l\pi \quad l \in Z$

V. $2\sin x = \operatorname{tg} x$ $\quad x \neq \frac{\pi}{2} + k\pi \quad k \in Z$

$2\sin x = \frac{\sin x}{\cos x} \quad /\cdot \cos x$

$2\sin x \cdot \cos x = \sin x$

$2\sin x \cdot \cos x - \sin x = 0$

$\sin x \cdot (2\cos x - 1) = 0$

Egy szorzat akkor és csak akkor nulla, ha valamelyik szorzótényező nulla.

$\sin x = 0$

$x = k\pi$

$2\cos x - 1 = 0$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$x_1 = 60° \cdot \frac{\pi}{180} + 2k\pi$

$x_2 = 300° \cdot \frac{\pi}{180°} + 2l\pi$

VI. Felhasználjuk a következőket

$\sin\alpha = \sin\beta$ akkor és csak akkor, ha a) $\alpha = \beta + 2k\pi$ $k \in Z$

b) $\alpha = \pi - \beta + 2k\pi$

$\cos\alpha = \cos\beta$ akkor és csak akkor, ha a) $\alpha = \beta + 2k\pi$

b) $\alpha = 2\pi - \beta + 2k\pi$

$\sin 2x = \sin\left(x + \frac{\pi}{3}\right)$

a) $2x = x + \frac{\pi}{3} + 2k\pi \quad / -x$

$x = \frac{\pi}{3} + 2k\pi$

b) $2x = \pi - \left(x + \frac{\pi}{3}\right) + 2k\pi$

$2x = \pi - x - \frac{\pi}{3} + 2k\pi$

$3x = \pi - \frac{\pi}{3} + 2k\pi$

$3x = \frac{3\pi - \pi}{3} + 2k\pi \quad /:3$

$x = \frac{2\pi}{9} + \frac{2k\pi}{3}$

$\cos\left(x + \frac{\pi}{4}\right) = \cos\left(x - \frac{\pi}{3}\right)$

a) $x + \frac{\pi}{4} = x - \frac{\pi}{3} + 2k\pi \quad / -x$

$\frac{\pi}{4} \neq -\frac{\pi}{3} + 2k\pi$

Nincs megoldás

b) $x + \frac{\pi}{4} = 2\pi - \left(x - \frac{\pi}{3}\right) + 2k\pi$

$x + \frac{\pi}{4} = 2\pi - x + \frac{\pi}{3} + 2k\pi$

$2x = 2\pi + \frac{\pi}{3} - \frac{\pi}{4} + 2k\pi$

$2x = \frac{24\pi + 4\pi - 3\pi}{12} + 2k\pi$

$x = \frac{25\pi}{24} + k\pi$

VII. $\sin x \geq \frac{\sqrt{2}}{2}$

$45° \cdot \frac{\pi}{180} + 2k\pi \leq x \leq 135° \cdot \frac{\pi}{180} + 2k\pi$

$\operatorname{tg} x < -1$

$-\frac{\pi}{2} + k\pi < x < -\frac{\pi}{4} + k\pi$ vagy $\frac{\pi}{2} + k\pi < x < \frac{3\pi}{4} + k\pi$

I a) $\sin\alpha = 0{,}993$

$7\sin x = 3$

$2\sin x = -\sqrt{2}$

b) $\cos x = -0{,}95$

$3\cos x = 2$

$\cos^2 x = 1$

c) $\operatorname{tg} x = 2{,}3$

$7\operatorname{tg} x = 9$

$\operatorname{tg} x = -1$

II. $\sin 5x = \frac{1}{2}$ $\qquad \cos 3x = 1{,}2$ $\qquad \operatorname{tg} 2x = 2$

$\sin\left(4x + \frac{\pi}{6}\right) = 1$ $\qquad \cos\left(x - \frac{\pi}{3}\right) = \frac{1}{2}$ $\qquad \operatorname{tg}\left(x + \frac{\pi}{4}\right) = -3$

III. $4\sin^2 x + 2\sin x - 1 = 0$

$2\cos^2 x + \cos x - 2 = 0$

$5\operatorname{tg}^2 x + 6\operatorname{tg} x = 11$

$\operatorname{tg} x - \operatorname{ctg} x = 2$

IV. $4\cos x = \operatorname{tg} x$

$\cos x - \sin^2 x = -0{,}4$

$\sin x \cdot \operatorname{tg} x = 1{,}5$

V. $3\sin x = 2\operatorname{tg} x$

$7\cos x = 4\operatorname{ctg} x$

VI. $\sin\left(x + \frac{\pi}{3}\right) = \sin 3x$

$\cos 2x = \cos\left(3x - \frac{\pi}{4}\right)$

$\sin\left(x - \frac{\pi}{5}\right) = \sin\left(x + \frac{\pi}{4}\right)$

VII. $\sin x \geqq \frac{1}{4}$ $\qquad \cos 2x \geqq 0$ $\qquad 3\operatorname{tg} x > 6$

$\sin x < -\frac{1}{3}$ $\qquad 2\cos < -1$ $\qquad \operatorname{tg} 4x \leqq \frac{1}{2}$